

Initial Conditions and Global Event Properties from Color Glass Condensate

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Abstract

Perturbative unitarization from non-linear effects is thought to deplete the gluon density for transverse momenta below the saturation scale. Such effects also modify the distribution of gluons produced in heavy-ion collisions in transverse impact parameter space. I discuss some of the consequences for the initial conditions for hydrodynamic models of heavy-ion collisions and for hard “tomographic” probes. Also, I stress the importance of realistic modelling of the fluctuations of the valence sources for the small- x fields in the impact parameter plane. Such models can now be combined with solutions of running-coupling Balitsky-Kovchegov evolution to obtain controlled predictions for initial conditions at the LHC.

Keywords: Heavy-ion collisions, Gluon saturation, Hydrodynamics, Initial Conditions

1. Introduction

The concept of perturbative saturation of the gluon density of a hadron at small light-cone momentum x was introduced originally to preserve unitarity of the scattering amplitude at high energy [1] which is violated by linear perturbative QCD. In particular, non-linear processes such as “gluon recombination” should prevent a power-law divergence of the (unintegrated) gluon density $\Phi(x, k_\perp^2)$ at small intrinsic transverse momentum k_\perp , and instead lead to its “saturation”: $\Phi(x, k_\perp^2) \sim \log 1/k_\perp^2$.

McLerran and Venugopalan suggested that for a large nucleus and at small x that the problem of gluon saturation could be addressed by classical

methods [2]. In their model, the high density μ^2 of valence charges per unit transverse area on the light cone acts as a source for a classical Yang-Mills field of soft gluons. This classical field, indeed, was shown to exhibit saturation for $k_\perp^2 < Q_s^2 \sim \alpha_s^2 \mu^2$ [3]. Most important for the present purposes, however, is that perturbative gluon saturation also predicts a modified distribution of small- x gluons in transverse impact parameter space: in the unitarity limit the local density of gluons, integrated over k_\perp^2 , is no longer simply proportional to the density of valence sources as expected in the dilute, linear regime.

The emerging consequences for the initial conditions for hydrodynamic models of heavy-ion collisions were realized later [4, 5]. Consider a collision of two heavy ions at non-zero impact parameter. Neglecting fluctuations of the local density of participants, their overlap area in the transverse plane has a short axis, parallel to the impact parameter, and a long axis perpendicular to it. If the produced gluons equilibrate then the pressure gradients convert this asymmetry of the initial density profile into a momentum asymmetry called “elliptic flow”, $v_2 \sim \langle \cos 2\phi \rangle$. In the absence of any scales (such as the freeze-out temperature T_f , the phase transition temperature T_c , or a non-vanishing mean free path λ) hydrodynamics predicts that v_2 is proportional to the eccentricity ε of the overlap area [6], $\varepsilon = \langle y^2 - x^2 \rangle / \langle y^2 + x^2 \rangle$. The average is taken with respect to the distribution of produced gluons in the transverse x - y plane.

A simple initial condition assumes that by analogy to the Glauber model for soft particle production $dN/dy d^2r_\perp \sim \rho_{\text{part}}^{\text{ave}}(r_\perp) \equiv (\rho_{\text{part}}^A(r_\perp) + \rho_{\text{part}}^B(r_\perp))/2$, where ρ_{part}^i is the density of participants of nucleus i per unit transverse area. A $\sim (5 - 20)\%$ contribution of hard particles needs to be added in order to fit the centrality dependence of dN/dy ; their transverse density scales like $\sim T_{AB}(r_\perp)$.

High-density QCD (the “Color-Glass Condensate”) predicts a different distribution of gluons in the transverse plane, corresponding to a higher eccentricity ε for intermediate impact parameters. In particular, when either A or B is dense the number of produced particles is proportional only to the density of the dilute collision partner, whose partons add up linearly. Hence, *in the reaction plane*, $dN/dy d^2r_\perp \sim \min(Q_{s,A}^2, Q_{s,B}^2) \sim \min(\rho_{\text{part}}^A, \rho_{\text{part}}^B)$ drops more rapidly towards the edge than $dN/dy d^2r_\perp \sim \rho_{\text{part}}^{\text{ave}}$ [5]. Thus, a higher eccentricity is a generic effect due to a dense target or projectile. Specific numerical estimates at the limited available RHIC energy do depend on the model for the unintegrated gluon distribution, however.

2. Modelling fluctuations of the large- x valence sources

Numerical estimates for the density distribution of produced partons require detailed modelling of the fluctuations of the large- x sources in impact parameter space. For peripheral collisions, and for smaller nuclei such as Cu, this is obvious. However, *differences* of moments of the density distribution, such as the eccentricity ε , exhibit sensitivity to fluctuations even for central collisions of heavy nuclei [7]. This is due to the fact that in the presence of fluctuations $\varepsilon \neq 0$ for central $b \rightarrow 0$ collisions (with x and y directions defined via the principal axes of the particle production zone), while it would otherwise vanish. The same applies to another moment, the “triangularity” [8], which gives rise to $\sim \langle \cos 3\phi \rangle$ contributions to the azimuthal distribution of particles (a non-vanishing triangularity arises in the CGC framework away from midrapidity, due to evolution of $Q_s^{A,B}$ with y , even without fluctuations of the sources [5]).

To model the fluctuations of the valence sources for the small- x fields in the transverse plane one notes that several distinct transverse distance scales are involved. The radius R_A of a nucleus is much larger than the radius R_N of a nucleon (the confinement scale) and one may therefore treat their fluctuations classically; in other words, we consider a collision of two “bags” of nucleons (quite densely packed, though) instead of using a quantum mechanical wave function for the nucleus. The transverse coordinates of the nucleons can be sampled randomly from a Woods-Saxon distribution. Multi-particle correlations are usually neglected (see, however, ref. [9]) except for a short-distance hard core repulsion which enforces a minimal distance ≈ 0.4 fm between any two nucleons.

The scale where particle production occurs within the CGC framework is $\sim 1/Q_s$. It is again much smaller than the radius of a nucleon,

$$\frac{1}{Q_s} \ll R_N \ll R_A , \quad (1)$$

and so the fluctuations of the valence charge density within a nucleon could again be treated independently from particle production. However, in practice nucleons are usually treated as hard spheres with uniform density.

Once a configuration of valence charges in the transverse plane has been obtained by Monte-Carlo methods, one constructs the unintegrated gluon distribution (the small- x fields) $\Phi(x, k_\perp^2; r_\perp)$ at every point r_\perp in the transverse plane [10]. The large- x valence charges can still be treated as frozen

sources since their fluctuations in the transverse plane occur over time scales much larger than the small- x evolution. In the model of Kharzeev, Levin and Nardi [11], for example, the unintegrated “Weizsäcker-Williams” gluon density of a nucleus (per unit transverse area) is given by

$$\Phi(x, k_{\perp}^2; r_{\perp}) \sim \frac{1}{\alpha_s} \frac{Q_s^2(x; r_{\perp})}{\max(Q_s^2(x; r_{\perp}), k_{\perp}^2)} . \quad (2)$$

$Q_s^2(x; r_{\perp}) = Q_0^2(r_{\perp}) (x_0/x)^{\lambda}$ exhibits the growth of the saturation momentum with energy expected from quantum evolution. In turn, $Q_0^2(r_{\perp})$ corresponds to the density of valence charge (squared) at the scale x_0 and needs to be determined for each configuration individually.

The fluctuations in the distribution of the hard sources in the transverse plane should not be confused with their distribution in color space. In the CGC framework, the color charge density per unit area of a nucleus or hadron is a stochastic variable, and all observables need to be averaged over its distribution. In the MV model, for example, this distribution is a *local* Gaussian,

$$W[\rho] \sim \exp \left(- \int d^2 r_{\perp} \frac{\text{tr } \rho^2(r_{\perp})}{\mu^2(r_{\perp})} \right) . \quad (3)$$

Since there are infinitely many points in a transverse domain on the order of a *fluid cell* (linear dimension $\sim 1/T$) these MV “fluctuations” of the valence charge density do not correspond to fluctuations of the initial condition (particle density) for hydrodynamics. While quantum evolution in x modifies the MV weight functional to a non-local distribution, the correlation length nevertheless remains of order $1/Q_s$ and therefore much smaller than the size of a fluid cell. Fluctuations of the evolution ladders should therefore have a small effect on the hydrodynamical evolution and on “global” event properties (v_2 etc.).

3. Applications

Fig. 1 shows the elliptic flow v_2 measured in heavy-ion collisions at RHIC scaled by the eccentricity ε of the overlap zone [12]. As already mentioned above, in the absence of any scales such as a non-zero mean free path, v_2/ε would be independent of the transverse density of particles. Indeed, if the v_2 data is scaled by the eccentricity obtained from a CGC implementation based on the KLN model then the required breaking of scale invariance is

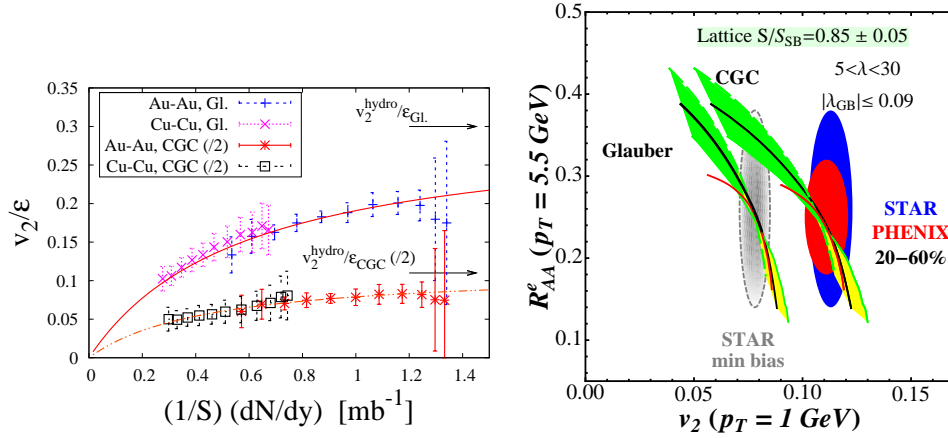


Figure 1: Left: v_2/ϵ versus the transverse density [12]; $v_2/\epsilon_{\text{CGC}}$ has been scaled by 1/2 for better visibility. Right: Correlation between the suppression of hard non-photonic electrons and v_2 of bulk particles expected from a gravity dual model [17].

seen to be lower than for Glauber-like initial conditions. Actual solutions of viscous hydrodynamics (for v_2) appear to confirm this simple observation in that the *slope* of v_2/ϵ versus transverse density is sensitive to the distribution of produced particles [13]. State-of-the-art simulations with realistic QCD equation of state and with initial conditions which account for the above-mentioned fluctuations will provide further insight.

Other recent studies [8, 14] have shown that fluctuations in the initial state which evolve through the hydrodynamic expansion can give rise to various structures observed in two-particle angular correlations, such as the “near-side ridge” or the “away-side cone”. The fact that these correlations are very long range in rapidity provides a link to CGC physics of heavy-ion collisions [15]: the classical color fields introduced with the MV model are boost invariant, while quantum evolution in $\log 1/x$ diminishes correlations only over rapidity intervals exceeding $\sim 1/\alpha_s$.

The modification of the distribution of produced particles in r_\perp space in the non-linear regime affects not only the hydrodynamical evolution but also energy loss of hard probes. This observable provides a “tomographic probe” of the density distribution of the bulk. The higher eccentricity of CGC-like initial conditions increases the azimuthal asymmetry of high- p_\perp jets [16]. On the other hand, at fixed multiplicity dN/dy and v_2 of bulk particles it reduces the suppression of hard non-photonic electrons expected from a gravity dual

model, c.f. fig. 1 (right) [17].

4. Future developments

Present CGC based estimates of the initial conditions for hydrodynamics from fluctuating valence sources usually rely on simple KLN-like models for the x , k_\perp and r_\perp dependence of the unintegrated gluon distributions (uGD). To improve the reliability of the approach one should incorporate constraints resulting from the measured (centrality dependent) p_\perp -distributions in d+Au collisions at central and at forward rapidity. Furthermore, to actually test the theory of small- x evolution it is important that extrapolations to LHC energy are based on uGDs which truly solve those evolution equations. Significant progress has been made recently in solving the running-coupling Balitsky-Kovchegov equation [18], and those results can now be combined with the MC codes which generate configurations of large- x valence sources [10].

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